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Applications and Improvements of Elo Mechanism

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Abstract
With the rapid development of the Internet, an increasing number of video game players appear, especially those with Multiplayer Online Battle Arena(MOBA) and First Person Shooter (FPS) games. This article is aimed to do an in-depth investigation of the Elo system in the popular MOBA games. Through the usage of quantitative and qualitative methods and, to address the current problem that Elo system isn't fair enough in video games' matching systems. Some potential improvements are given to solve some aspects of this problem.

Keywords
Elo; Mathematical theory; MOBA games

Introduction

In order to create a fairer matching and rating mechanism, and improve the gaming experience, Elo system is introduced, which was originally widespread from the category of chess competitions to basketball tournaments, and currently, with the appearances and developments in computer games, it's presently using in the matching mechanism for MOBA and FPS games. It has been a long history of this system used in fields of sport.

Elo based his metric on one previously used by the United States Chess. Federation (USCF), which was measured relative to the performance of an "average" player in the United States Open Championship (Glickman and Jones, 1999), then people affirmed the fairness and objectivity of this approach, but the experiences showed the chess players' performances didn't follow a normal distribution, instead of that, logistic distributions were used to find their expected winning probability, and that has been shown by the results came from official games' relating websites.

However, the matching systems of most

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computer games output the opposite feedback. For instance, many players oppose the matching system in one Chinese game: Honor of Kings(and there is a similar foreign one called Arena of Valor), and the only two reasons for this are: opponents are too strong and teammates are too weak.

Nowadays, game companies usually use only one point, which is known as the ’hidden point’ of players to show their game level, and a hexagon is given to them to show their six-game metrics. The problems caused by these are:

1. The point tends to be different according to the variance in places, for example, Attack Damage Carry(ADC) usually gets higher marks in destroying turrets and damaging, while supports always get more assists and bearing damages.

2. Players with higher skills tend to have the same, or even lower winning probability, compared with ones with lower skills. The current Elo system mixes them, to make high-skilled players lead low-skilled ones to play, causing them to have almost equal winning chances.

In this paper, quantitative methods (questionnaires with a sample size of approximately 120, most of the candidates are university students), and qualitative ones (detailed interviews with 3 expert players of high-school level) are proposed, which mainly focus on MOBA games’ playing experiences, in order to show the difference of regarding games between game levels, education levels, and positions they play.

Literature Review

Mathematical Theory

Kovalchik (2020) et al. proposed a model using the normal distribution for competitor’s rating at the start of each period. However, from Sismanis (2010) et al. an idea was proposed that the Elo system uses a single rating per player to predict the result of a match by fitting a logistic curve to the differences in the players’ ratings, which has become an important output for later researches. Aebischer (2017) et al.

proposed that the Elo rating system is based on the probability of winning or losing in a confrontation, as determined by a logistic distribution, and just like the research in 2016, Wheatcroft (2020) et al. proposed that the Elo rating system assigns a single rating to each participating player or team based on their overall ability, using logistic regression, proved the result for the research in 2016. Finally, logistic distribution in Elo system came up by Ebtekar and Liu (2021) et al. They claimed that Elo system robustness turns out to be a natural outcome of precisely predicting performances using heavy-tailed distributions, such as the logistic ones.

Guardiola and Natkin (2005) et al. pointed out that game theory is a tool for understanding scenarios in which agents make decisions and the interplay of tactics is stated as gain, and it is frequently used in conjunction with other fields of mathematics such as statistics, probability, and linear programming, which was a precedent for researching in this area and made a great impact on the later research. Then synoptically, Fritsch, Voigt, and Schiller (2008) et al. proposed that the problem of inflation in the Elo System can be mitigated through appropriate mathematical adjustments (reduction of a constant value in the formula for rating computation). In the same year, Moreland and Superdock (2018) et al. proposed that the technique in Elo system is fundamentally Bayesian, with prior and posterior information of each team’s performance reflected in rating values before and after the match. Ultimately, Xiong, Yang, Zin, and Iida (2016) et al. proposed that Elo style had been not only a way of ranking and matching, but it could also be the basic system for several other ranking systems, which have been developed and utilized in communities or organizations.

Related Applications and Games

In the category of chess competitions, Glickman and Jones (1999) et al.mentioned that Elo rating system's most important feature is that it allows competitors of all levels to track their development as they improve their chess skills. In addition, Gerdes and Gränsmark (2010) et al. came up with that the introduction of the Elorating, which allowed chess players' strengths to be compared on a metric scale, was a watershed moment in the evolution of chess as an analytical tool. Finding the best move in a chess position is a highly complex, genuine human activity, and each chess position represents a well-defined problem environment, with a set amount of identifiable moves that can be played at any given point – ideal for studying search processes and problem-solving.

Applications in MOBA Games

The original product phase of League of Legends(LoL), like the idea from

Myślak and Deja (2014) et al. that one of the reasons that LoL succeeded is (mainly Summoner's Rift) gave lots of different game modes, and the targets are different with time in a match. More in detail, Nascimento Junior, Melo, da Costa, and Marinho (2017) et al. noted that MOBA games are extremely competitive due to the tremendous diversity and dynamicity of players' in-game actions as well as their performance, e.g. gold won, dead champions, damage done, healing received, etc.

Because of the game's popularity and tournaments, this competitiveness is magnified in LoL. Similarly, Yang et al. (2022) et al. proposed that one factor to estimate the winning rate is pre-game elements including hero choosing and hero roles, the other one is team kills, heroes' experience, and gold, which are all available in real-time (during the match). Let the position of an assassin as an example, Cheng et al. (2019) et al. proposed that assassins abuse more since abusive players choose assassins rather than being abusive while choosing assassins to demonstrate that a team is not made up of separate players.

With the massive points in order to win, the games would be fairer and there would be little probability for players to win these games just by luckiness.

On the other hand, Kho, Kasihmuddin, Mansor, Sathasivam, et al. (2020) et al. noted that, as the level of competition rises, so does the demand for strategies to improve players' performance. For example, Ontanón et al. (2013) et al. proposed that Real-time Strategy is a strategy game sub-genre in which players must gather resources, establish bases, and military forces in order to overcome their opponents (destroying their army and base), with simultaneous and durative moves. Players must coordinate plans, tactical movements, reconnaissance missions, itemization synergies, and resource sharing amongst themselves after being placed together from a pool of several million (Ferrari, 2013). Due to the large number of temporary teams in LoL, success in the game necessitates not only competent in-game talent but also vital social skills (Kou and Gui,2014).

At the same time, Pramono, Renalda, and Warnars (2018) et al. pointed out that to attain those results, collecting more player information and selecting which information may be relevant to their talent and how they contribute to the team are both crucial to take it into account for a better and fair matchmaking system to get better matchmaking results, and with the massive game strategies and the Elo system for matching and ranking, the divisions are divided separately. Kou, Gui, and Kow (2016) et al. proposed that Bronze division(about 40.77 % of players), Silver (37.54 %), Gold (14.38 %), Platinum (5.99 %), Diamond (1.22 %), Master (0.05 %), and Challenger (0.02 %) are the tiers in LoL.

Matching System

In terms of matching players, Duersch, Lambrecht, and Oechssler (2020) et al.proposed that the Elo system has the advantage of adjusting players' ratings not only based on the outcome but also based on the strength of their opponents. It also has the ability to incorporate learning. For instance, Pelánek (2016) et al. proposed a model that the update of points in Elo system is small when strong players beat weak players. As a result, only a few rating points will be taken from the low-rated player if the high-rated player wins (Edelkamp, 2021). Therefore, the advantages of Elo are gradually obvious. Firstly, it modifies the players' ratings based on not just the game outcome but also the players' ratings before the game. (Szczecinski and Djebbi, 2020), and in addition, it can be employed for matchmaking when we witness a battle of players with nearly the same ability level inspiring the entertainment component of hosting a tournament (Makarov, Savostyanov, Litvyakov, and Ignatov, 2017).

Method

This work has been carried out using mainly two groups of methods: qualitative and quantitative studies. We tried to use both techniques in order to not only look at the numbers and study statistics, but also try to find the deep reasoning behind the player's psychological behaviour, and then decided on what we could potentially improve the current Elo system.

Quantitative Study

The quantitative study mainly used the data collected from the questionnaires. In the questionnaires, there were questions about self-evaluations, objective and subjective markings, and research about players' income and daily gaming time.

There were about 120 people who filled out the questionnaires in the two-week collecting time during the Chinese New Year 2022. All of the candidates were selected from MOBA game players, from novice ones to expert ones, and the questionnaire was dropped in many WeChat groups, which consisted of mainly university students. After collecting players' responses from the questionnaire, the normal distribution test and histogram in the 'explore' section in Statistical Product and Service Solutions(SPSS) were used to test for which distribution the data are represented.

Qualitative Study

In the qualitative study, we used the interview of players with years of experience in games, including MOBA games. We had informal face-to-face interviews with three expert game players, Player 1, Player 2, and Player 3, with challenger divisions and at least 2 years of playing MOBA and FPS games, who gave specific and important information about their feelings of ranking in these games in interviews. Player 1 is an expert in position Jungle and he was proud of his ability to destroy his opponent's base secretly in his 3-year career, while Player 2 played his best as a Top Laner and ADC, he was an extremely confident player, not only in games with superiority but also in inferior ones. The achievement gained by him could exist in all the competitions. Player 3 was good at playing

Support and Mid Laner with supporting type, it was quite difficult for him to achieve in Master Division with individual ranking in the era that players discriminated against Supports. The questions for an interview were set to get more information about their psychologies and behaviours while winning or losing streak as well as their views of the hexagon representation of players' skills and the fairness of the matching system.

Results

In this paragraph, we include selected visualization of our processed data, along with some of the very indicative tables and the corresponding explanation. We tend to give a very clear indication of how this our research is carried out and how this could be potentially applied to some of the other applications to improve the existing Elo system from a different perspective. The results include the numbers with the visualization showing the outcome from the questionnaires, and this is mainly from the quantitative method. The detailed discussion combining the quantitative and qualitative results to give a more global conclusion will be shown in the analysis and evaluation section.

The results here used various methods, from basic statistics, including pie charts and simple line charts to see the trend, to the more advanced data processing using SPSS, including normal distribution tests, etc. We would like to know how each different perspective could potentially affect the psychology of gamers while there is an Elo system.

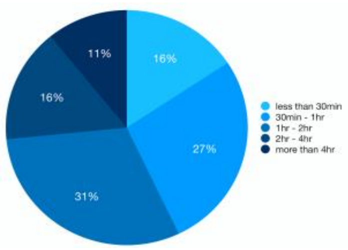

Figure 1 . Average time for players spending on games daily

Figure 1. shows the average daily time used in playing all the video games collected from players, from never to more than 4 hours.

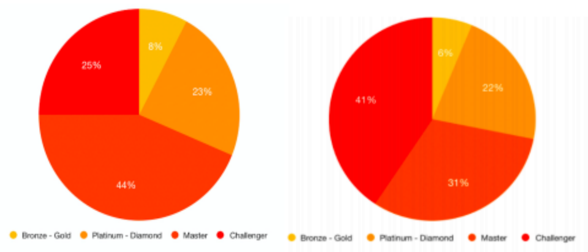

Figure 2. The average division and the highest divisions achieved by players

Figure 2. (left) shows the average division achieved in ranking in MOBA games, collected from MOBA game players, from Bronze to Challenger Division. The (right) other one shows the highest division achieved in ranking in MOBA games, collected from MOBA game players, from Bronze to Challenger Division.

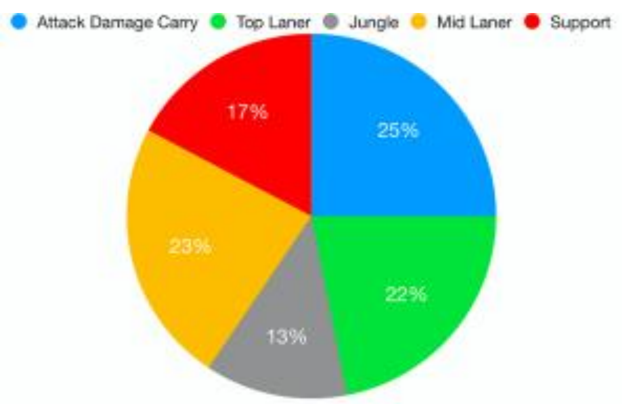

Figure 3. Percentage of the best position played by players

Figure 3. shows the best position player in MOBA games, among all the positions: ADC, Top Laner, Jungle, Mid Laner, Support, collected

from MOBA game players.

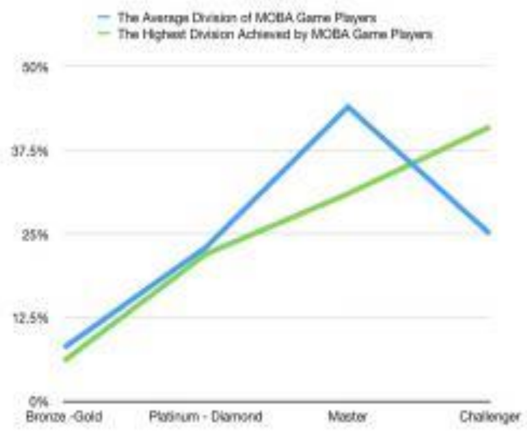

Figure 4. Percentage of players in each division

Figure 4. shows the relationship and distributions between average divisions and highest divisions in playing MOBA games, collected from MOBA game players.

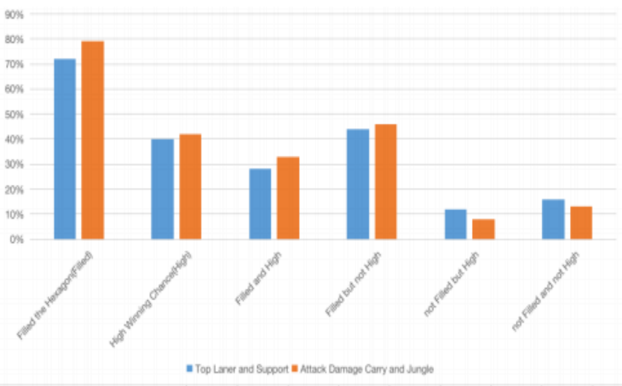

Figure 5. Whether the hexagon filled relates to a high winning chance

Figure 5. collects the information in two groups of positions in the MOBA games: top layer with support and ADC with jungle. We ordered the figure using a bar chart from left to right in the order of filled to unfilled hexagon, and from high to low winning chance.

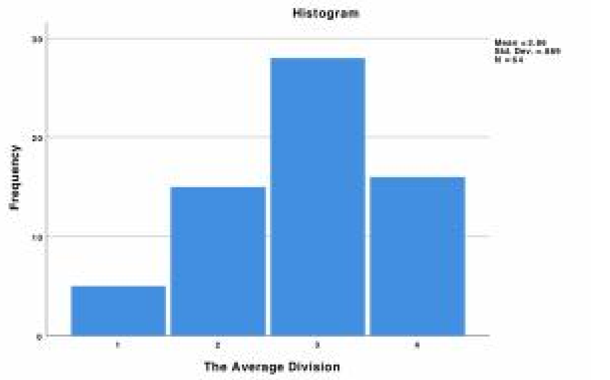

Figure 6. Histogram of The Average Division of Players

Figure 6. collects the information from MOBA game players to show their average division in all competition seasons. (Bronze to Gold is represented by 1, Platinum to Diamond is represented by 2, Master is represented by 3, Challenger is represented by 4.)

Descriptives

			Statistic	Std. Error
The Average Division	Mean		2.86	.111
	95% Confidence Interval for Mean	Lower Bound	2.64	
		Upper Bound	3.08	
	5% Trimmed Mean		2.90	
	Median		3.00	
	Variance		.789	
	Std. Deviation		.889	
	Minimum		1	
	Maximum		4	
	Range		3	
	Interquartile Range		2	
	Skewness		-.417	.299
	Kurtosis		-.486	.590

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
The Average Division	.250	64	<.001	.861	64	<.001

a. Lilliefors Significance Correction

Figure 7. The Normal Test for Average Division

Figure 7. collects the average divisions from players and calculates the features of data, like median, mean, and skewness, using SPSS.

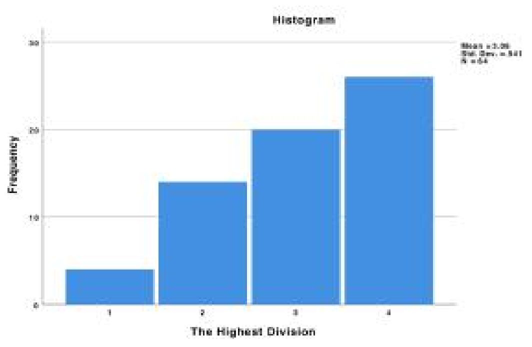

Figure 8. Histogram of The Highest Division of Players

Figure 8. distributes the highest division achieved by players collected in the interview results. Because the abilities of candidates usually outperformed other players, most candidates were in the Master and Challenger Divisions (Division 3 and 4).

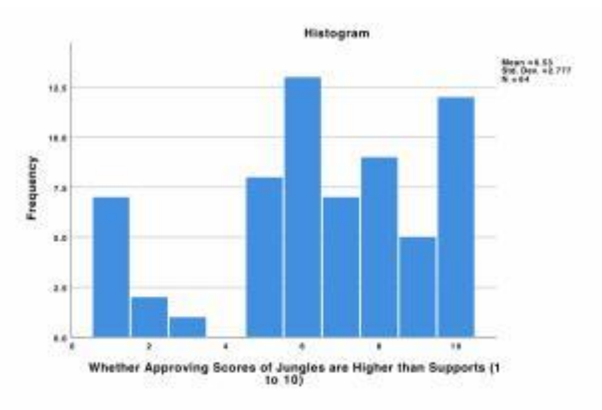

Figure 9. Results of Whether Approving Scores of Jungles are Higher than Supports (0 to 10)

Figure 9. distributes the result of whether it's correct that the average scores of jungles are higher than the ones of supports. (From 0: disagree to 10: agree).

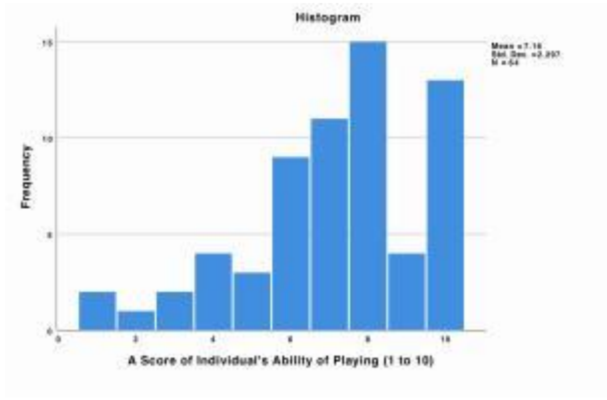

Figure 10. Results of Self- evaluations

Figure 10. shows the result collected for the self-evaluations of candidates, between 0 marks: weakest and 10 marks: strongest.

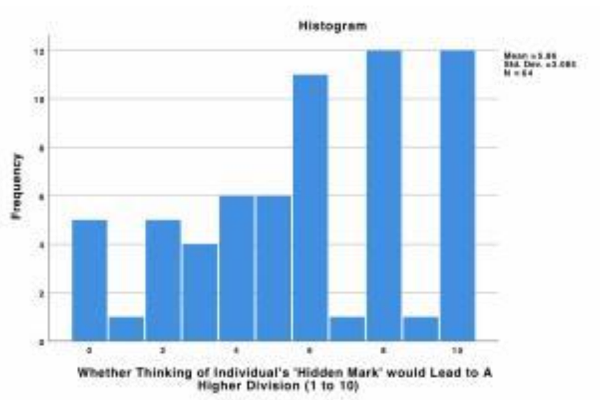

Figure 11.Results of judging whether players 'hidden marks' can lead them to a higher division

Figure 11. shows the result collected about whether players think their 'hidden marks' can lead them to a higher level (but for some reason the division isn't gained), with 0 to 10 marks for disapproving to approving.

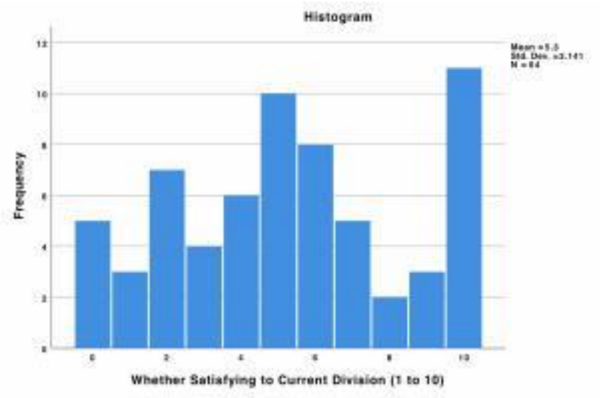

Figure 12. Results of marking the level of satisfaction for players' current divisions

Figure 12. collects the results of the level of satisfaction for players' current divisions, with 0 (low satisfaction) to 10 (very satisfied).

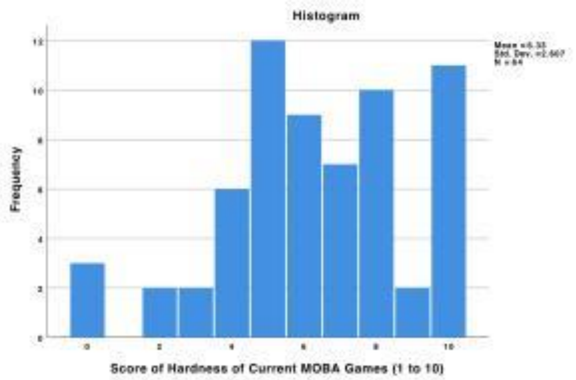

Figure 13. Results of players' judgments of the hardness of the MOBA games they played

Figure 13. shows results of marking the assumptive hardness of current MOBA games among candidates, with 0 marks for extremely easy to 10 marks for absolutely hard.

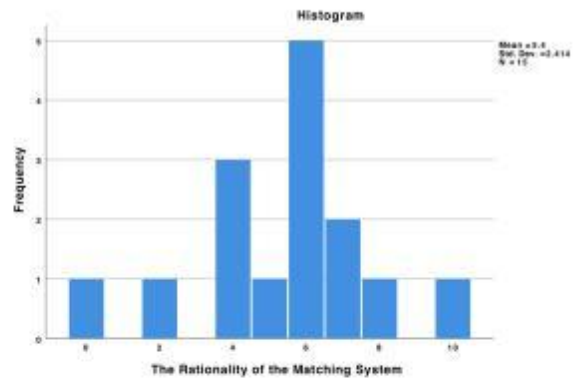

Figure 14. The judgment of whether a current matching system is rational

Figure 14. collects the scores marked by candidates, showing the rationality of the current matching system, from the questionnaire, with 0 marks: not rational, to 10 marks: fully rational.

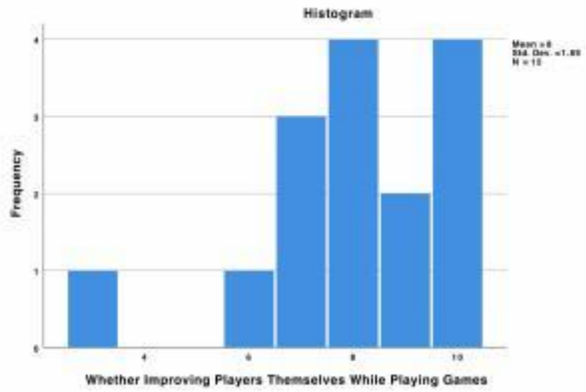

Figure 15. The result of whether players tend to improve their game skills while playing games

Figure 15. distributes the tendency of players to improve themselves while playing games, with 0 marks: only enjoy the happiness of games, to 10 marks: practicing a game skill is the most important thing to do in games.

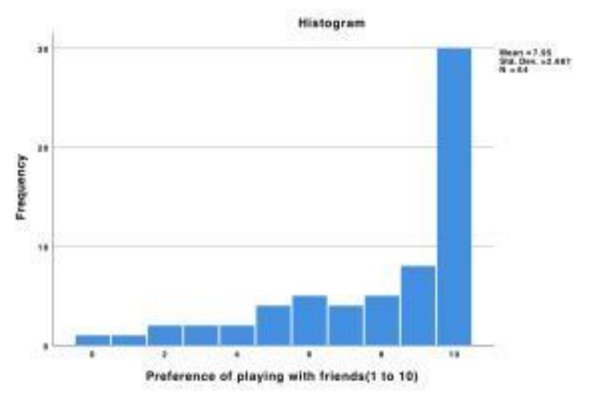

Figure 16. Results of players' markings to whether they would like to with their friends

Figure 16. shows the distribution of players' preference of playing with friends, with 0: like to play alone, to 10: most likely to play with friends.

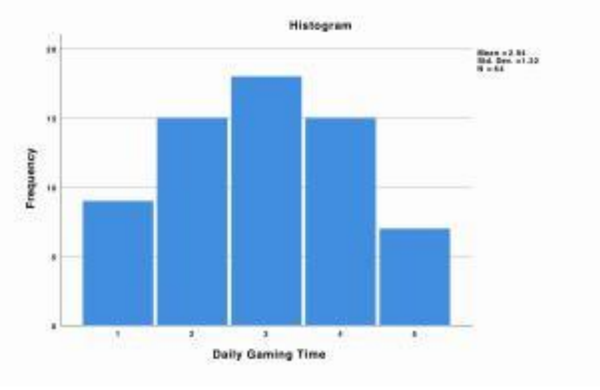

Figure 17. Players' daily gaming time

Figure 17. shows the average daily gaming time among players, with index 1: less than 30 minutes, 2: 30 minutes to 1 hour, 3: 1 hour to 2 hours, 4: 2 hours to 4 hours, and 5: more than 4 hours.

Descriptives

			Statistic	Std. Error
Daily Gaming Time	Mean		2.94	.152
	95% Confidence Interval for Mean	Lower Bound	2.63	
		Upper Bound	3.24	
	5% Trimmed Mean		2.93	
	Median		3.00	
	Variance		1.488	
	Std. Deviation		1.220	
	Minimum		1	
	Maximum		5	
	Range		4	
	Interquartile Range		2	
	Skewness		.014	.299
	Kurtosis		-.908	.590

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Daily Gaming Time	.154	64	<.001	.915	64	<.001

a. Lilliefors Significance Correction

Figure 18. Normal test of daily average time

Figure 18. collects the daily average time of playing games from players and calculates the features of data, like median, mean, and skewness, using SPSS.

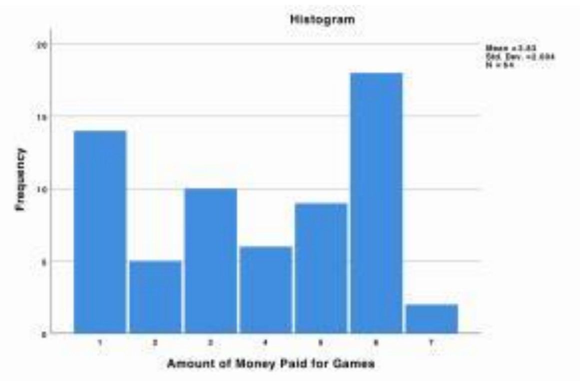

Figure 19. The total amount of money paid for games among the candidates

Figure 19. collects the total payments for games by the questionnaire, with index 1: less than 10 (RMB yuan), 2: 10 to 50, 3: 50 to 100, 4: 100 to 500, 5: 500 to 1000, 6:1000 to 10000, and 7: more than 10000.

Figure 20. Levels of monthly salary of candidates

Figure. 20. shows the distribution of candidates' monthly salary, with index 1: less than 1000 (RMB yuan), 2: 1000 to 4000, 3. 4000 to 10000, 4: 10000 to 50000, 5: more than 50000.

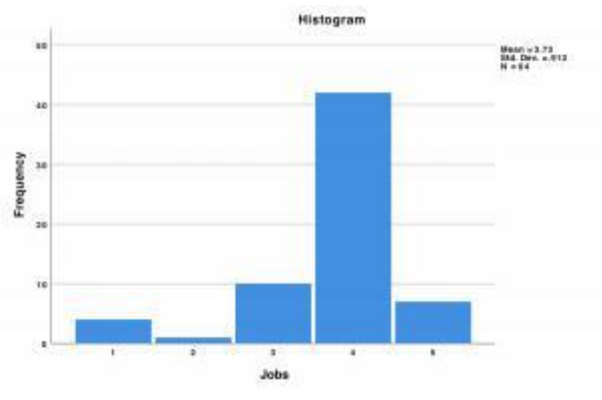

Figure 21. Education levels of candidates

Figure 21. shows the distribution of education levels of players collected from the questionnaire, with index 1: Primary school, 2: Middle school, 3: High school, 4: University, and 5: At work.

Analysis and Evaluation

Analysis

Quantitative Analysis

Starting from the quantitative methods, firstly, we can see that the data in Figure. 2 show that most players who filled the questionnaire were in the Master and Challenger division, and the relationship between the number of players and their average divisions is represented closely to the normal distribution, as for Figure. 7. In the 'tests of normality for this diagram, we only need to see the Kolmogorov-Smirnov test for the number of values greater than 50. Df value shows there are 64 valid answers between 1(Bronze to Gold) to 4(Challenger), and if the Significance is less than 0.01, which is less than the critical value of 0.05, then accept H0, that is, there is no evidence to show the distribution is not normal. However, if players got onto a greater division, the same change would be shown for their highest division. As a result, 25% of players got Challenger division for almost every competition season, and there were 16% more who rarely had achieved this height of division.

In addition, there was an even distribution showing the best position played by players, as shown in Figure.3. More specifically, the percentage of players who played the least popular position, Jungle, was only 12% less than playing the most popular one, ADC. In this questionnaire, we divided the game positions into 2 parts: Support with Top Laner, and ADC with Jungle, which represents taking damage and damaging, respectively, and gathered data that can show the relativity between filling the game-skill hexagon and winning chance, between the two groups of positions. Figure. 5 shows players were not satisfied with their current winning chances overall, even though they thought the hexagon had been filled out by themselves, and the overall relationship is similar between the two groups of positions, but fortunately, the current Elo matching system seems to be fair for it led to a balance between winning and losing. Moreover, some questions about grading to express players' views, with scores of 1 to 10, were placed in the questionnaire. Generally, a higher mark means more 'agree' or 'better' than lower marks. For instance, Figure. 10 shows players' self-evaluation, and most of them gave themselves 8 points, which is a middle case, and for whether satisfying their current division, most of the candidates made good aspects of their current division (Figure. 12), and most of them showed great desire to advance (Figure. 11). As a result, they tended to improve themselves, and work hard to fit the Elo matching system (Figure. 15) with their friends (Figure. 16).

Ultimately for the game itself, players marked a score about the hardness of current MOBA games, and another score about the rationality of the current matching system. Most of them thought the current game is of medium to difficult hardness (Figure. 13), and the rationality was judged to be medium hard (Figure. 14), as well.

Finally, as for the time spent on games and money paid for them, the results of daily gaming time tend to be represented in a normal distribution (Figure. 17) and the descriptive, as well as normal tests, are shown in Figure. 18. In the 'tests of normality for this diagram, we also only need to see the Kolmogorov-Smirnov test for the number of values greater than 50 again. However, even if the Significance in the Shapiro-Wilk test is less than 0.01, then we also need to accept H0, to ensure that is in a normal distribution. Besides this, the skewness of these two normal tests can also show their normal distribution as their values are both approximate to zero. That shows no observable deviations are shown in the diagrams. As for the percentage check in Figure. 1, most players play for 30 minutes to 2 hours a day, some of them may be suffered from the health system, which is a Chinese policy in order to prevent teenagers addicting to games. There's also a remarkable point that more than 50% of candidates were students and had no income (Figure. 20 and Figure. 21), but only 22% of players said they had never paid for every content of games, and surprisingly, 28% players, which occupies the highest fraction of the result, each of them had spent 1000 to 10000 yuan on games (Figure. 19). That is an alarming truth that the Elo system indirectly affected the paying habits for teenagers, it also claimed the necessity of setting policy of restricting them to play video games.

Qualitative Analysis

Combining the results from the qualitative study, in the 3 interviewees' words, beating their opponents harshly, being praised by their teammates, and getting into a higher division can gain their game satisfaction, and all of them approved that the hexagon in games to show players' skills is not enough and depends mostly on players' positions. For instance, jungles are more likely to fill the 'money' part but it's almost impossible for supports to reach that level. Fortunately, they summarized their ways to adjust to the inevitable Elo matching system, like using weak heroes while finding it's quite hard to win the game, in a ban/pick turn. The best mechanism is to let players truly match at random instead of relying on others to win, and currently, the matching and ranking mechanism is fair, but not with equality. Every player enjoys the joy of winning, but these can be inflated, and it would be inequitable to players with real strength.

Ultimately, forcing players to increase their gaming time is intolerable, according to their interview results. An example of this is quoted by Player 3. "Game companies created a fair and rational game mechanism, but they didn't optimize the current foul game environment. In the current Elo system, the visible hexagon leads to an absurd rating system, which alters players' hidden points' with no reason, and that makes some positions that proposed to take damage, especially supports, quite hard to gain their divisions. " Player 2 also claimed, "The best game mechanism is no mechanism existing, and random matching seems to be a better mechanism than large numbers of complex algorithms. " Currently, with the existence of power-leveling services and 'game actors', individual game experiences are affected to be worse. Therefore, as Player 2 had mentioned, totally random could really be a better matching mechanism than any other formulas.

Conclusion

To conclude, I've found the main drawbacks and potential improvements in the current Elo system, as well as factors about matching in ranking games. For example, the hexagon in games should be multi-dimensional, and cover more different periods in each game, and according to the results gathered in qualitative studies(interviews) and quantitative ones(questionnaires), different places tend to have different degrees of filling in the hexagon and varying winning chances. Furthermore, in future studies, the number of experimental subjects, which means, the number of candidates doing the questionnaire and interview will be increased, to make fairer and more accurate results, and the distribution will be shown more clearly after using SPSS to analyze. In addition, getting to know the deeper algorithm of Elo in order to improve the fairness mathematically is an important project to research and study, and it will be complementary to existing research.

The overall experiment investigated the current Elo system and some potential improvements based on statistics and in-depth interviews. From the data collected, we mainly visualized it so that the cognitive cost is reduced to find the pattern hidden behind those data. Further research could be done by carrying out machine learning (ML) methods to deal with those data, mainly for classification tasks as well as regression modeling. By combining the data and the comments from domain experts (those players), we had a thorough investigation of the design of the Elo system as well as some of the other effects that could potentially affect the gaming experience. As e-sports are considered more and more as a formal competition, we feel the importance of bringing those improvements to cope with the new gaming system, so that the 'score' for each candidate could therefore be measured accurately and fairly.

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